Super-Quadratic Quantum Speed-Ups and Guessing Many Likely Keys

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Key Guessing Problem

Given: oracle access to some function $f_k : K \to \{0,1\}$ with

$$f_k(x) = \begin{cases} 1, & x = k, \\ 0, & \text{else.} \end{cases}$$

Find: key k.

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Optimal classical algorithm:

- Iterate over key space K in random order.
- Expected runtime $= \dfrac{|K| + 1}{2}$.

Optimal quantum algorithm:

- Grover search.
- Expected runtime $\approx |K|^{1/2}$.

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Optimal cla

What if we know the probability distribution D of k?

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Find: key k.

Optimal cla

- Iterate over key space K in random order. Grover search.

- Expected

What if we know the probability distribution D of k?

Main result:

Super-quadratic quantum speed-up for every non-uniform D.

Key Guessing from a Distribution *D*

Optimal classical algorithm:

- Iterate over key space K in decreasing order of likelihood.

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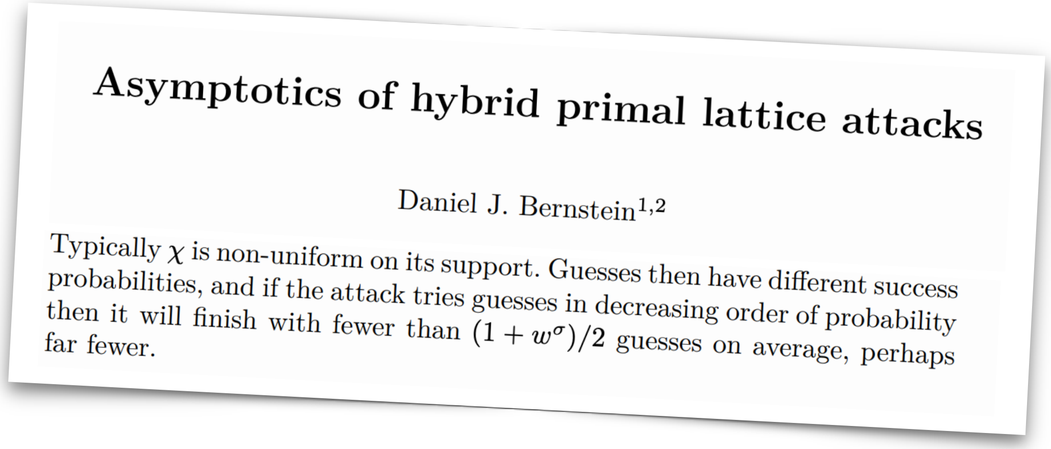

Asymptotics of hybrid primal lattice attacks

Daniel J. Bernstein[1,2]

Typically χ is non-uniform on its support. Guesses then have different success probabilities, and if the attack tries guesses in decreasing order of probability then it will finish with fewer than $(1 + w^\sigma)/2$ guesses on average, perhaps far fewer.

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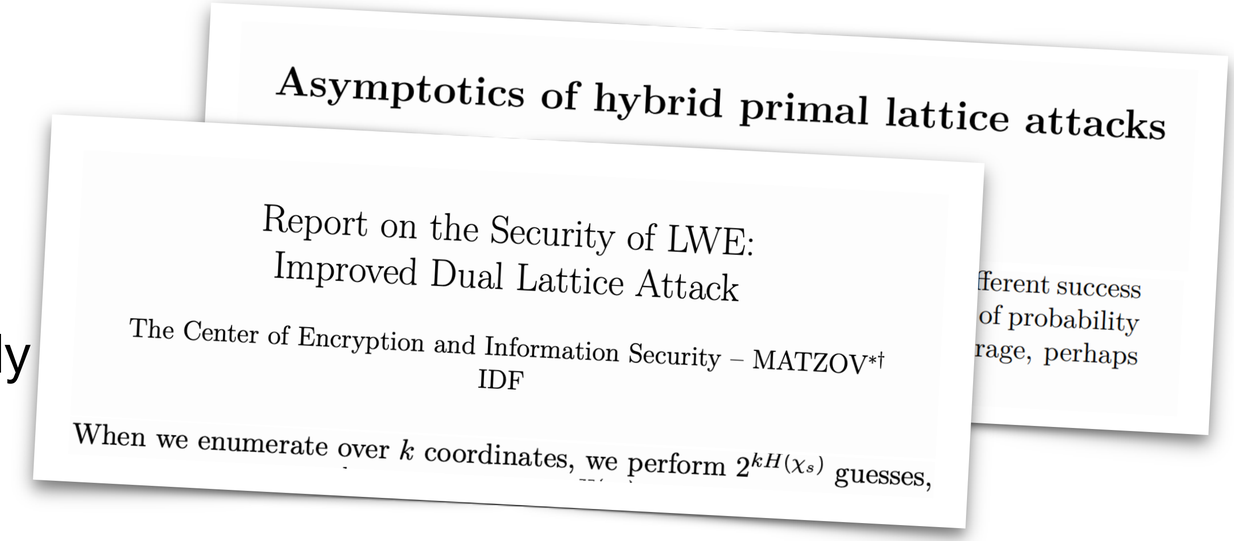
Asymptotics of hybrid primal lattice attacks

Report on the Security of LWE:
Improved Dual Lattice Attack

The Center of Encryption and Information Security – MATZOV*†
IDF

When we enumerate over k coordinates, we perform $2^{kH(\chi_s)}$ guesses,

fferent success
of probability
rage, perhaps

Key Guessing from a Distribution *D*

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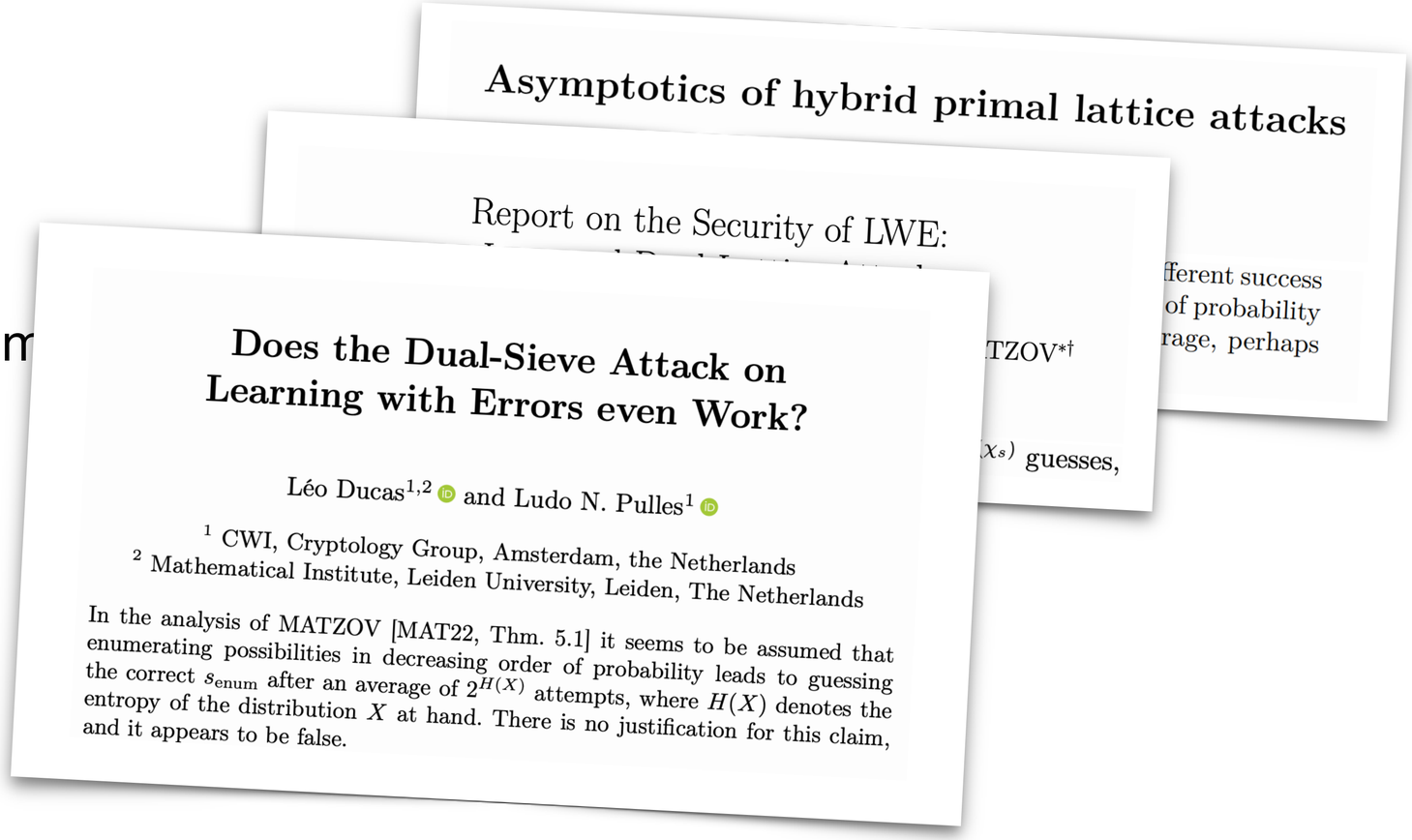
Asymptotics of hybrid primal lattice attacks

Report on the Security of LWE:

Does the Dual-Sieve Attack on Learning with Errors even Work?

Léo Ducas[1,2] and Ludo N. Pulles[1]

[1] CWI, Cryptology Group, Amsterdam, the Netherlands
[2] Mathematical Institute, Leiden University, Leiden, The Netherlands

In the analysis of MATZOV [MAT22, Thm. 5.1] it seems to be assumed that enumerating possibilities in decreasing order of probability leads to guessing the correct s_{enum} after an average of $2^{H(X)}$ attempts, where $H(X)$ denotes the entropy of the distribution X at hand. There is no justification for this claim, and it appears to be false.

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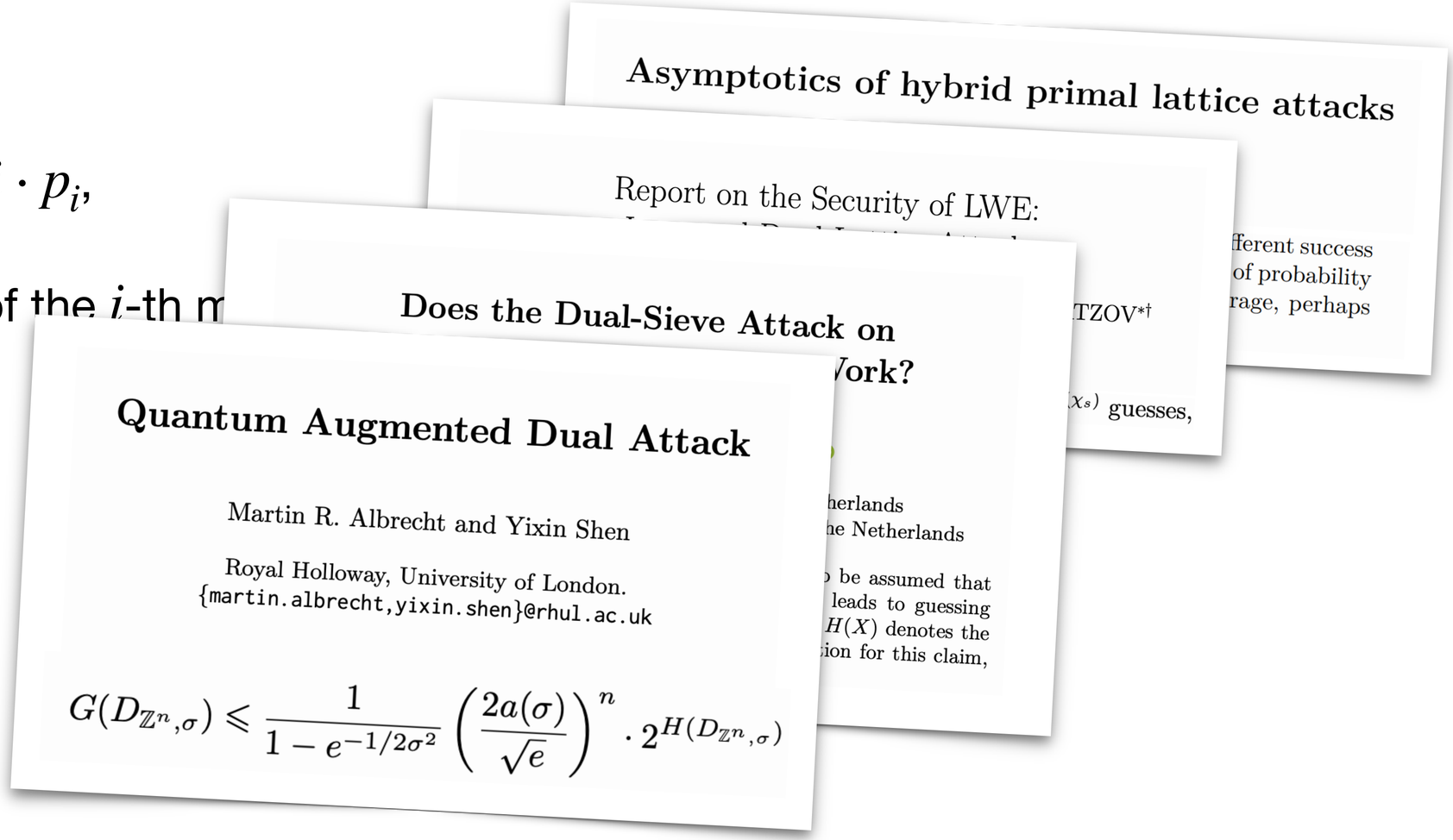
Asymptotics of hybrid primal lattice attacks

Report on the Security of LWE:

Does the Dual-Sieve Attack on

Quantum Augmented Dual Attack

Martin R. Albrecht and Yixin Shen

Royal Holloway, University of London.
{martin.albrecht,yixin.shen}@rhul.ac.uk

$$G(D_{\mathbb{Z}^n,\sigma}) \leqslant \frac{1}{1-e^{-1/2\sigma^2}} \left(\frac{2a(\sigma)}{\sqrt{e}}\right)^n \cdot 2^{H(D_{\mathbb{Z}^n,\sigma})}$$

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Key Guessing from a Distribution *D*

Optimal classical algorithm:

- Ite
- Ex

wh

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 42, NO. 1, JANUARY 1996 99

An Inequality on Guessing and its Application to Sequential Decoding

Erdal Arikan, *Senior Member, IEEE*

Improved Estimation of Key Enumeration with Applications to Solving LWE

Alessandro Budroni[1] and Erik Mårtensson[2,3]
Cryptography Research Center, Technology Innovation Institute, Abu Dhabi, UAE[1]
alessandro.budroni@tii.ae
Selmer Center, Department of Informatics, University of Bergen, Norway[2]
Department of Electrical and Information Technology, Lund University, Sweden[3]
erik.martensson@{uib.no,eit.lth.se}

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Rényi entropy with parameter 1/2

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Rényi entropy with parameter 1/2

Rényi Entropy

For a distribution D with support K and probabilities $p_1, \ldots, p_{|K|}$, we define

$$H_\alpha(D) := \lim_{\gamma \to \alpha} \frac{1}{1-\gamma} \log \left(\sum_{i=1}^{|K|} p_i^\gamma \right).$$

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Special cases:

- $H_0(D) = \log(|K|)$

  ☞Max entropy

- $H_1(D) = -\sum_i p_i \log(p_i)$

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Properties:

- D uniform

  $\implies H_\alpha(D) = H_\beta(D)$, for all $\alpha, \beta \in [0, \infty]$

- D non-uniform

  $\implies H_\alpha(D)$ decreases as α increases

- Product distributions $D = \chi^n$

  $\implies H_\alpha(D) = n \cdot H_\alpha(\chi)$

Quantum Key Guessing from a Distribution *D*

Optimal quantum algorithm:

- Sort keys in decreasing order of likelihood $k_1, k_2, \ldots, k_{|K|}$.
- Partition keys as
  - $K_0 = \{k_1\}$,
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Montanaro, TQC 2011

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Classical runtime:

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Quantum runtime:

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Arikan, 1996:

For every $\rho \geq 0$, we have $\sum_{i=1}^{|K|} i^{\rho} \cdot p_i \approx 2^{\rho H_{1/(1+\rho)}(D)}$

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Quantum runtime:

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Quantum speed-up: $2\frac{H_{1/2}(D)}{H_{2/3}(D)} > 2$

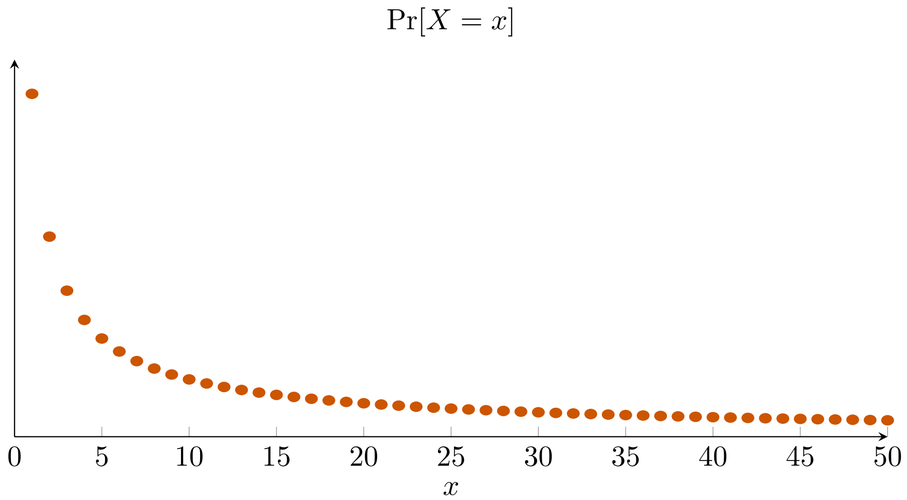
for every non-uniform D

Example: Zipf Distribution (Passwords)

Definition:

- Fix parameters N and t.
- $\Pr[X = x] \propto \dfrac{1}{x^t}$ for $x = 1, \ldots, N$.

- Leaked LinkedIn passwords (approximately) follow Zipf distribution with $N = 2^{27.2\cdots}$ and $t = 0.777$.



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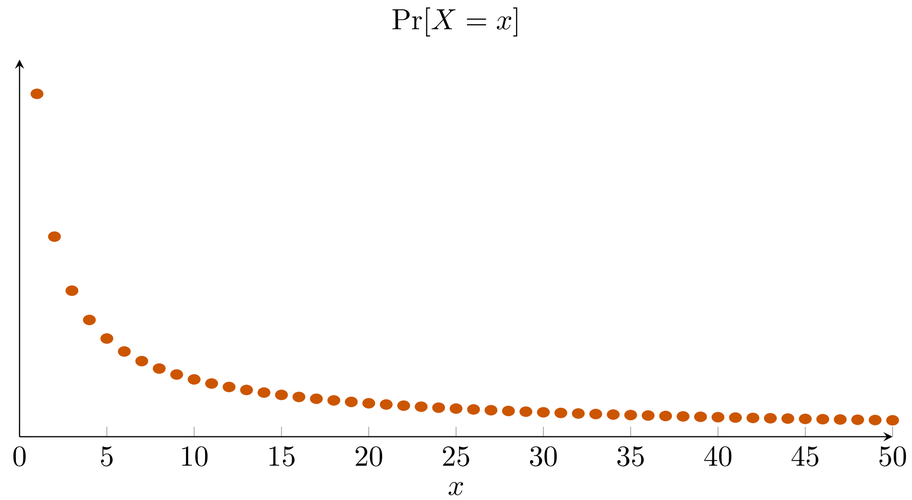

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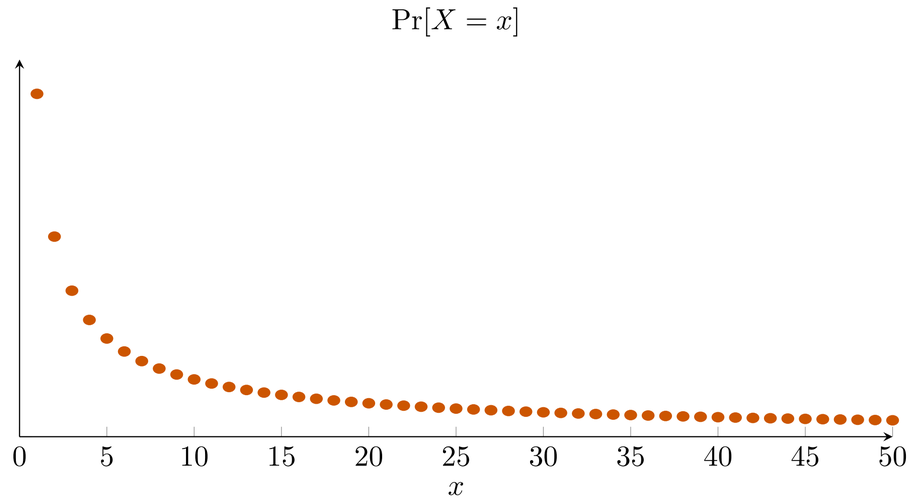

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Quantum guessing:

- $2^{H_{2/3}(D)/2} = 2^{13.1\ldots}$

Speed-up:

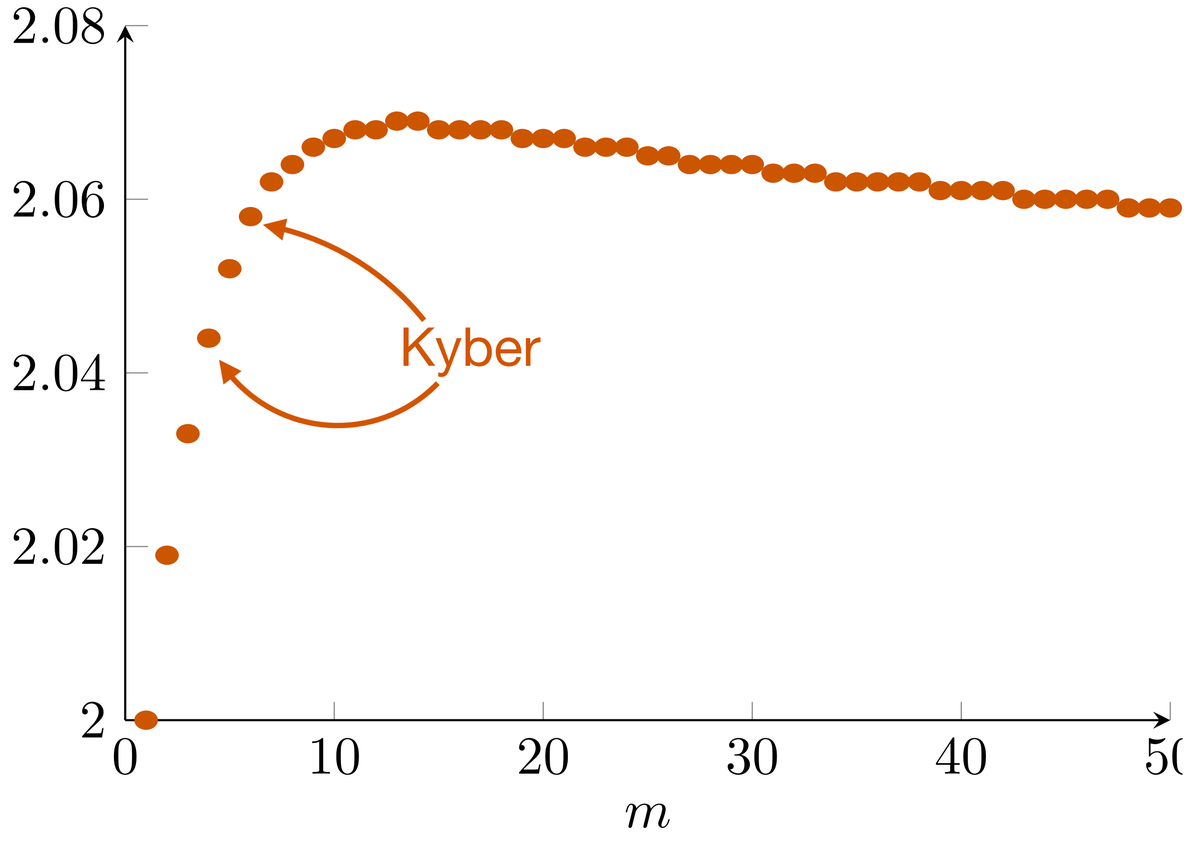
- 2.03

Example: Centered Binomial (Kyber)

Definition:

- Sample $(X_1, \ldots, X_m) \leftarrow \{0,1\}^m$ uniformly.
- Output $X_1 + \ldots + X_m - \frac{m}{2}$.

Quantum speed-up:



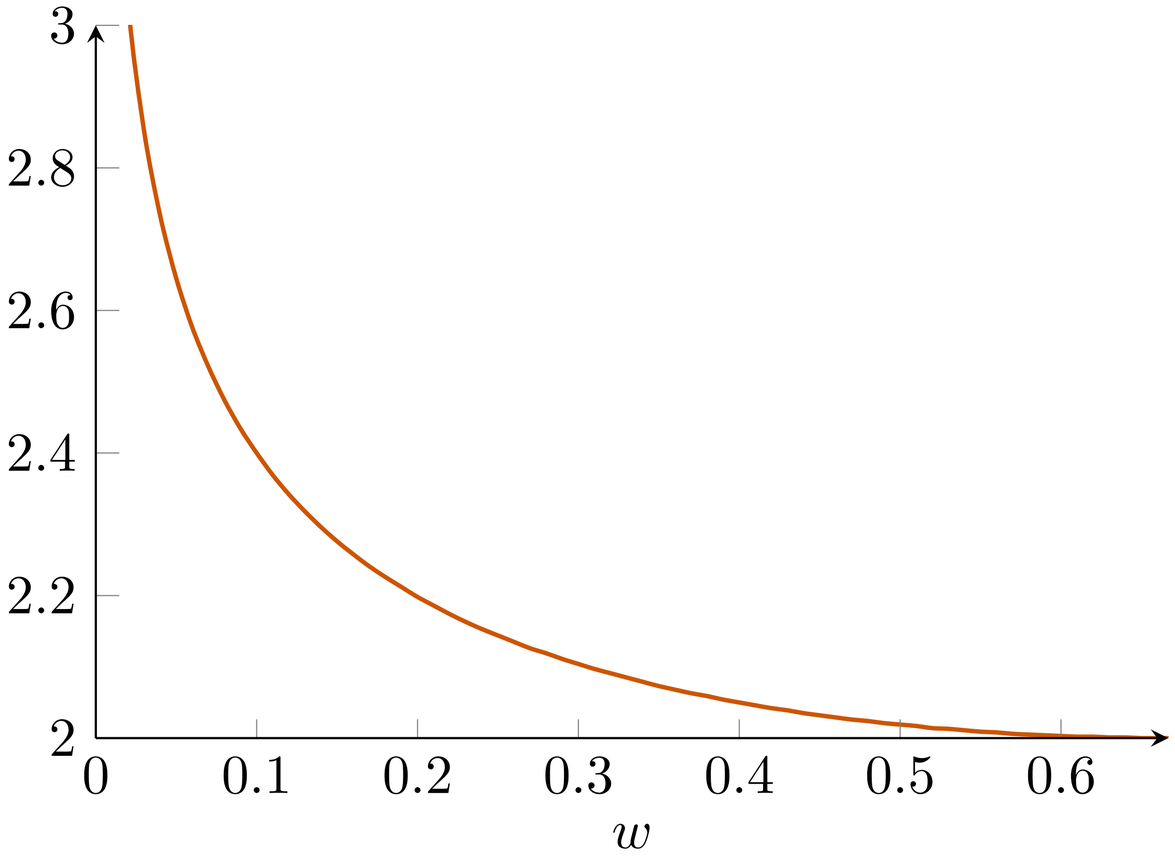
Example: Weighted Ternary (FHE)

Definition:

- $\Pr[X = 1] = w/2$ ☞$w \in (0,2/3]$
- $\Pr[X = -1] = w/2$
- $\Pr[X = 0] = 1 - w$

- Used for sampling ternary n-dimensional vectors of expected Hamming weight wn.

Quantum speed-up:



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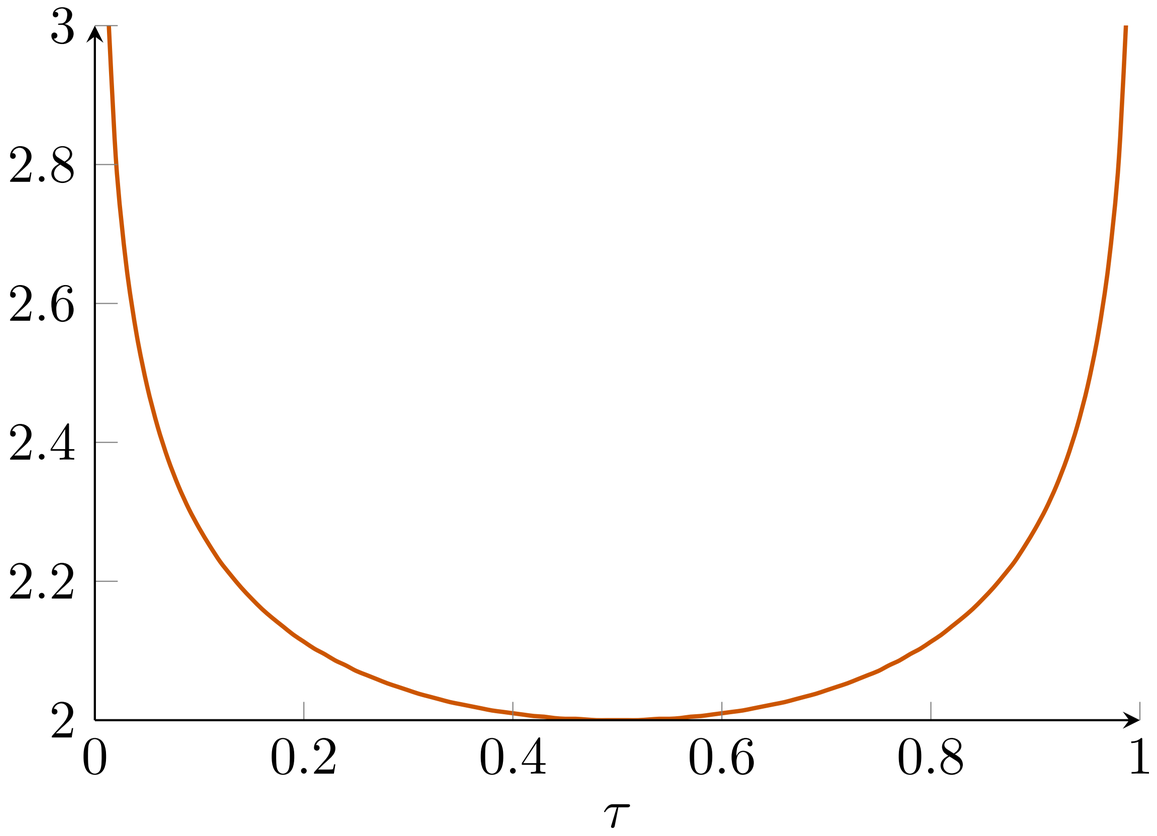

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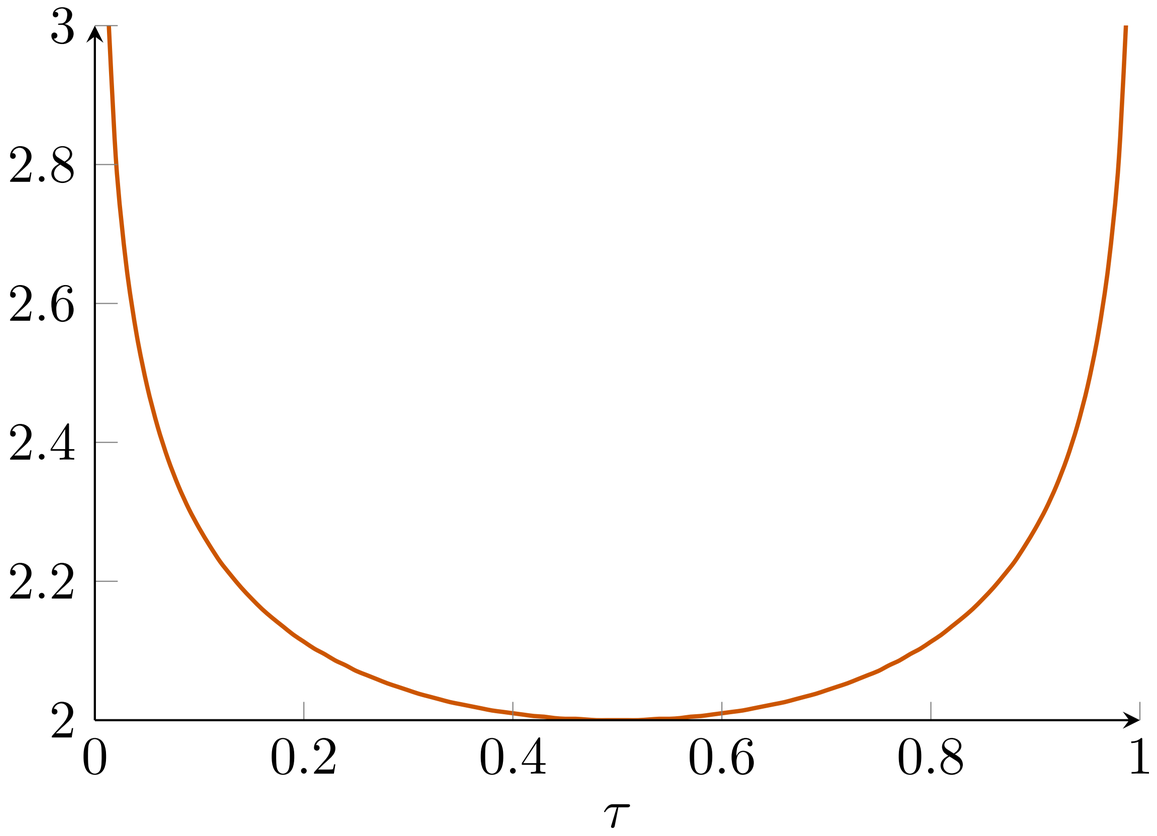
- LPN-based encryption uses n-fold Bernoulli with $\tau = n^{-1/2}$.
- Quantum speed-up: $\approx n^{1/12}$.

Quantum speed-up:



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Likely Keys

Example:

- LPN subkey drawn from n-fold Bernoulli distribution with $n = 15$, $\tau = 0.2$.
- Let T be RV for number of required guesses.

- Arikan: $\mathbb{E}[T] \approx 2^{H_{1/2}(D)} = 6476.64\ldots$

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- Markov's inequality: $T \gg \mathbb{E}[T]$ is unlikely.
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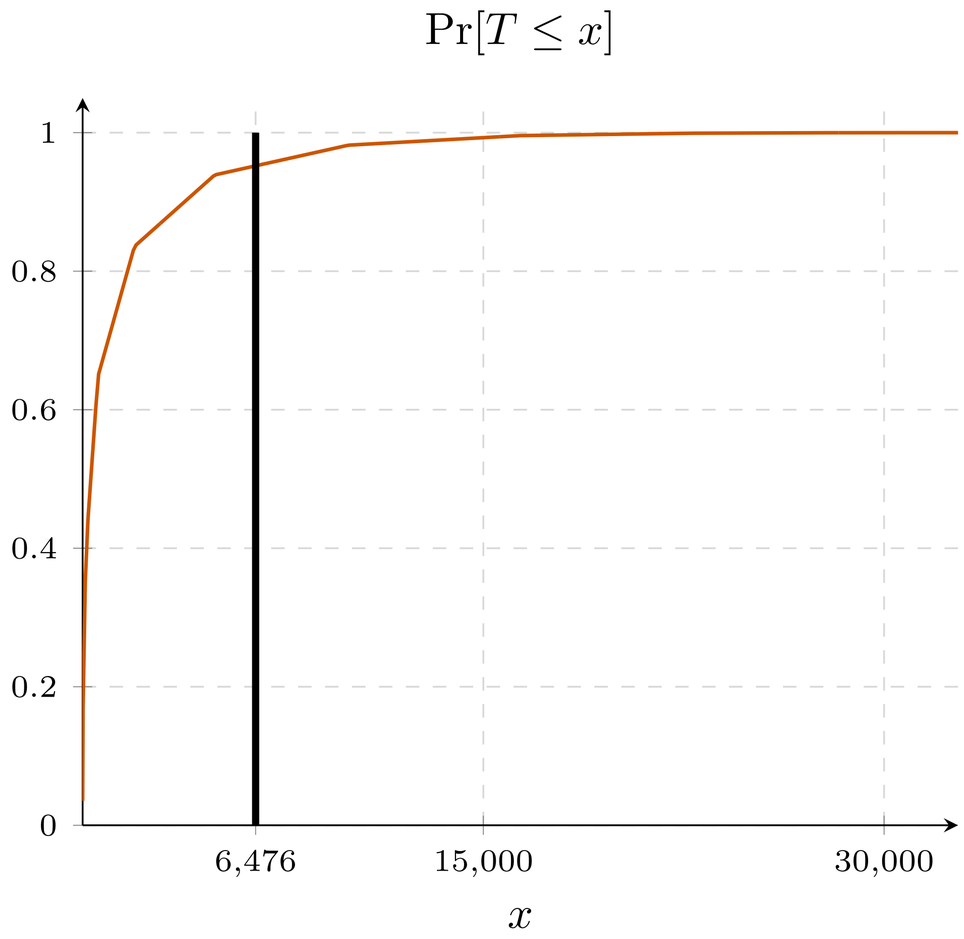

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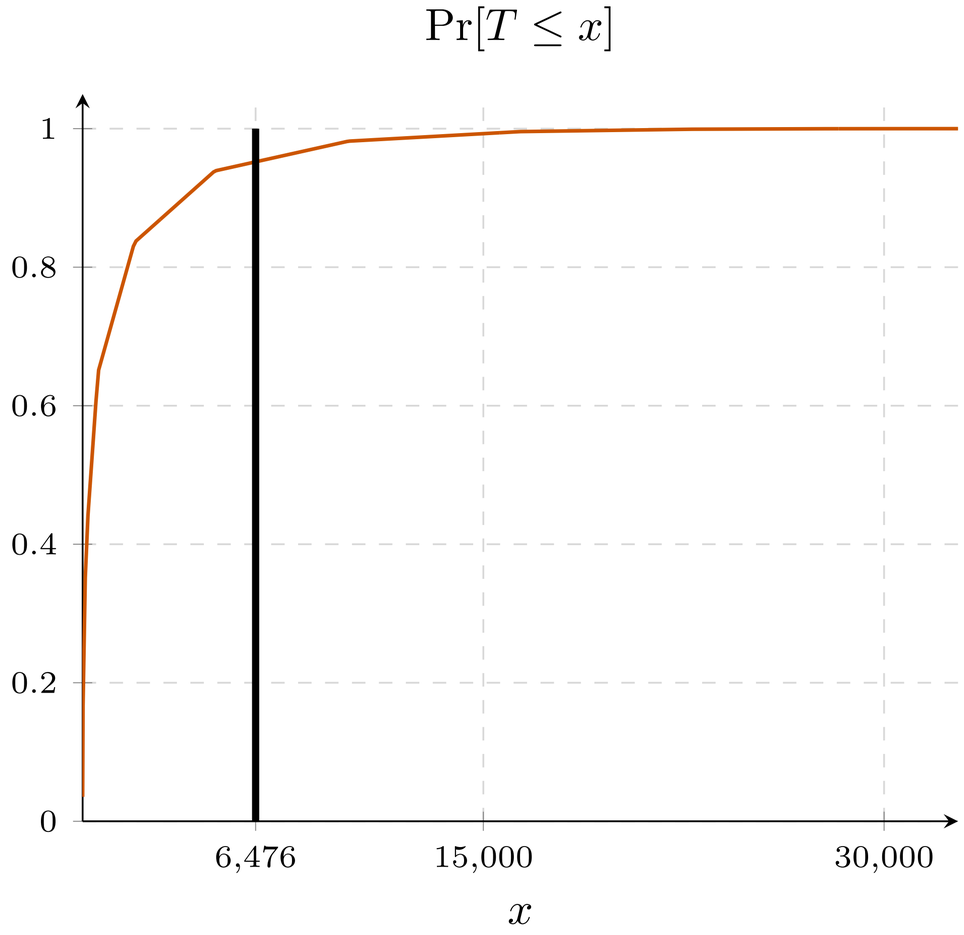

⚠ **32,768** possible keys.
307 most likely keys carry 50% of probability mass.

Beyond Expected Runtime

Alternative measures of complexity:

- ratio T/ε of runtime T and success probability ε
- expected time to find one out of m keys

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Results:

- T/ε is minimized when trying only the most likely key.

Beyond Expected Runtime

Alternative measures of complexity:

- ratio T/ε of runtime T and success probability ε
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Results:

- T/ε is minimized when trying only the most likely key. ☞ $T/\varepsilon = 2^{H_\infty(D)}$
- Upper bounds for one-out-of-m:

$$m \cdot 2^{H_{1-1/(m+1)}(D)}$$

classically

$$m \cdot \sqrt{2^{H_{1-1/(2m+1)}(D)}}$$

quantumly

Beyond Expected Runtime

Alternative measures of complexity:

- ratio T/ε of runtime T and success probability ε
- expected time to find one out of m keys

Results:

- T/ε is minimized when D is the most likely key. ☞ $T/\varepsilon = 2^{H_\infty(D)}$
- Upper bounds for one-out-of-m:

m	1	∞
classically	$2^{H_{1/2}(D)}$	$2^{H_1(D)}$
quantumly	$\sqrt{2^{H_{2/3}(D)}}$	$\sqrt{2^{H_1(D)}}$

$$m \cdot 2^{H_{1-1/(m+1)}(D)}$$

classically

$$m \cdot \sqrt{2^{H_{1-1/(2m+1)}(D)}}$$

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Is H_1 the *correct* measure of complexity?

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Lower bound for all classical algorithms

enumerating $\ll 2^{H_1(D)}$ candidates

$\Longrightarrow$ succ. probability ≈ 0

Algorithm matching the lower bound

sampling $2^{H_1(D)}$ times from D

$\Longrightarrow$ succ. probability $\gtrapprox 5/16$

$\ll$, $\approx$, $\gtrapprox$ depend on n

Follow-up Work: Impact for Lattices

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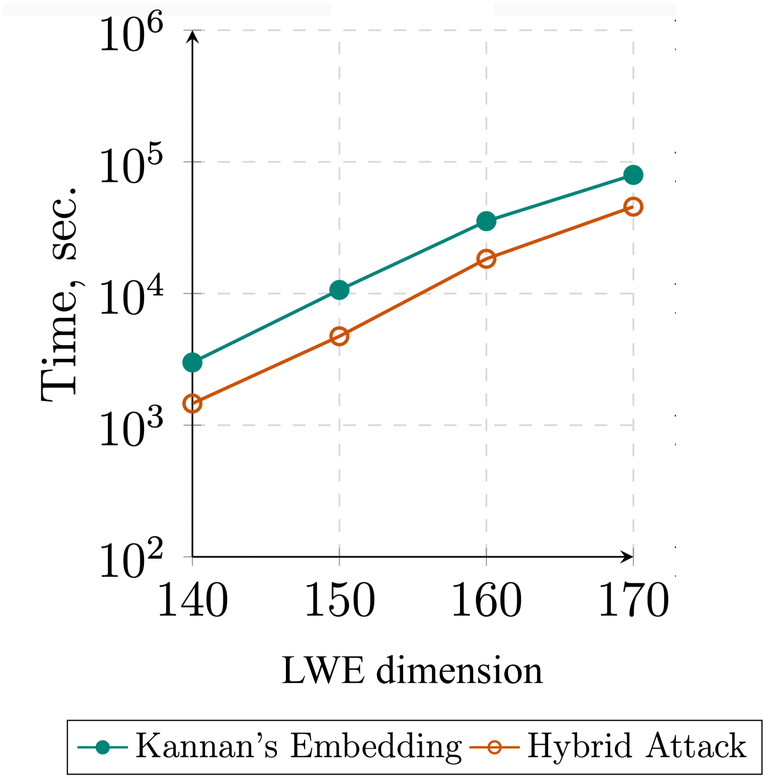
Practical performance on down-scaled Kyber



Summary

	Single Key	One-out-of-m
classically	$H_{1/2}(D)$	$H_{1-\frac{1}{m+1}}(D)$
quantumly	$H_{2/3}(D)/2$	$H_{1-\frac{1}{2m+1}}(D)/2$

https://ia.cr/2023/797